

Constrained Optimization (advanced course)

Abstract

In this course, we will study the fundamental theory and basic techniques for constrained optimization of multivariate functions. The course focuses mostly on continuous optimization, but the last chapter introduces some well-known methods for discrete optimization using stochastic heuristic techniques.

Pre-requisites

- Linear algebra
- Numerical methods (C omputo Cient ifico A)

Contents

- 1.- Introduction to optimization
 - 1.1.- Elements of optimization
 - 1.2.- Mathematical formulation
 - 1.3.- Continuous vs. Discrete optimization
 - 1.4.- Global vs. Local optimization
 - 1.5.- Stochastic vs. Deterministic optimization
 - 1.6.- Convexity
- 2.- Fundamentals of constrained optimization
 - 2.1.- Problem definition
 - 2.2.- Feasibility and solutions
 - 2.3.- Active constraints
 - 2.4.- Lagrangian
 - 2.5.- First-order optimality conditions (Karush-Kuhn-Tucker conditions)
 - 2.6.- Second-order conditions
 - 2.7.- Elimination of variables
- 3.- Linear Programming
 - 3.1.- Linear programs
 - 3.2.- Solutions of a linear program

- 3.3.- Optimality conditions
- 3.4.- Geometry of the feasible set
- 3.5.- The Simplex Method
- 3.6.- Implementation details for the Simplex Method

- 4.- Quadratic Programming
 - 4.1.- Equality-constrained quadratic programs
 - 4.2.- First-order necessary conditions
 - 4.3.- Solving the KKT system for a quadratic program
 - 4.4.- Schur-Complement method
 - 4.5.- Null-space method
 - 4.6.- Inequality-constrained quadratic programs
 - 4.7.- Active-sets methods
 - 4.8.- Quadratic sub-problems
 - 4.9.- Gradient projection for bound-constrained QPs

- 5.- Penalization methods
 - 5.1.- Introduction
 - 5.2.- Quadratic penalty for equality-constrained problems
 - 5.3.- Quadratic penalty for inequality constraints
 - 5.4.- Quadratic penalty algorithm
 - 5.5.- Nonsmooth exact penalty methods
 - 5.6.- Augmented Lagrangian methods

- 6.- Discrete stochastic optimization heuristics
 - 6.1.- Random search
 - 6.2.- Simulated annealing
 - 6.3.- Genetic algorithms

Bibliography

- Numerical Optimization. Jorge Nocedal and Stephen J. Wright. Springer.
- Introduction to Nonlinear and Global Optimization. Eligius M.T. Hendrix and Boglárka G.-Tóth. Springer.