GEOMETRY OF PERTURBATION THEORY FOR HAMILTONIAN SYSTEMS

JOSÉ A. VALLEJO

Resumen. Syllabus of a graduate course

Contents of the course

Hamiltonian systems. Lie and Poisson algebras. Examples. Derivations and Hamiltonian vector fields. Integral curves of Hamiltonian fields: Hamilton equations. Examples.

Mechanics on symplectic manifolds. The symplectic foliation of a Poisson manifold. Non-degenerate case. Cotangent manifolds and phase spaces. Integrable systems. Arnol'd-Liouville theorem and action-angle variables. Separability. Stäckel theorem. Canonical transformations and Hamilton-Jacobi theory. Examples: harmonic oscillator, Kepler problem, Hénon-Heiles system.

KAM theorem. Manifolds diffeomorphic to (product of) torus. Statement of the theorem. Main consequences. Proof of the theorem. The small denominators problem. Classical chaos.

Lindstedt-Poincaré theory. Formal series: main properties and methods of calculus. Secular perturbation theory. Lindstedt correction. Examples. Implementation of the method in a CAS.

Local normal forms. Lie series. Canonical perturbations through Lie-Deprit series. Canonical transformations and normal forms. Homological equations. Fourier analysis for systems whose non-perturbed flow is periodic. Averaging techniques. Deprit algorithm.

Global normal forms. Geometric interpretation of normal forms. Perturbation of vector fields. Global normal forms for Hamitonian systems admitting an $\mathbb{S}^1-$action. Algorithms for determining normal forms. Implementation in a CAS. Examples: elastic pendulum, Hénon-Heiles system. Particular study of slow-fast systems.

Symmetry reduction. Generating systems of Poisson algebras. Hopf variables. Reduced phase space: Hopf projection. Dynamics on reduced phase space. Critical points and resonances. Analysis through normal forms.

The quantum case. Basic notions on quantization. Metaplectic quantization. Normal forms of quantum Hamiltonian systems admitting an $\mathbb{S}^1-$action. Quantization of adiabatic invariants on slow-fast systems.

Referencias

[AM 87] R. Abraham, J. Marsden: *Foundations of Mechanics*, 2nd edition. Addison-Wesley (1987).

[AVV 1] M. Avendaño-Camacho, J. A. Vallejo and Yu. Vorobjev: *A simple global representation for second-order normal forms of Hamiltonian systems relative to periodic flows*. Journal of Physics A: Mathematical and Theoretical **46** (2013) 395201.

[AVV 1] M. Avendaño-Camacho, J. A. Vallejo and Yu. Vorobjev: *Higher order corrections to adiabatic invariants of generalized slow-fast Hamiltonian systems*. Journal of Mathematical Physics **54** (2013) 082704.

[BC 97] L. M. Bates and R. Cushman: *Global Aspects of Classical Integrable Systems*. Birkhäuser, Basel (1997).

[Cus 94] R. Cushman: *Geometry of Perturbation Theory*. In 'Deterministic Chaos in General Relativity'. NATO ASI Series Volume **332** (1994) 89–101.

[Kar 90] M. V. Karasev: *New global asymptotics and anomalies for the problem of quantization of the adiabatic invariant*. Funct. Anal. Appl. **24** 2 (1990) 104–114.

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