

ALGEBRAIC METHODS IN GEOMETRY AND PHYSICS

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ABSTRACT. Syllabus for a graduate course

CONTENTS

Manifolds as ringed spaces. Brief résumé of category theory. Rings and modules. Pre-sheaves and sheaves. Ringed spaces: examples. Differential and exterior calculus on ringed spaces.

Supermanifolds. Motivation: bosonic and fermionic calculus in quantum mechanics. \mathbb{Z}_2 -linear algebra and superalgebras. Lie superalgebras. Supermanifolds and Lie supergroups.

Lie algebroids. Vector bundles and morphisms between them. The notion of Lie algebroid: examples. The Gerstenhaber algebra associated to a Lie algebroid. Generating operators. Relation between Lie algebroids and supermanifolds. Lie-Poisson algebroids.

Differential operators on $\Omega(M)$. Canonical bracket on $\text{End}(\Omega(M))$. Derivations and differential operators. Tensorial operators. Operators that commute with the exterior differential d . Vector-valued differential forms and generalized Lie derivatives. The Frölicher-Nijenhuis decomposition.

Brackets. The Richardson-Nijenhuis bracket of vector-valued forms. The Frölicher-Nijenhuis bracket: the Nijenhuis torsion and characterization of integrable structures. The Schouten-Nijenhuis bracket of multi-vector fields. Super-Poisson structures. Omni-Lie structures.

Dirac structures. Distributions. The Courant bracket. The correspondence between Dirac structures and smooth pre-symplectic foliations. Dirac structures induced by Poisson brackets. Gauge transformations. Canonical actions and invariant Dirac structures. Hamiltonian actions.

REFERENCES

- [BBH 91] C. Bartocci, U. Bruzzo, D. Hernández-Ruipérez: *The Geometry of Supermanifolds*. Kluwer Academics (1991).
- [Cou 90] T. J. Courant: *Dirac manifolds*. Trans. Amer. Math. Soc. **319** 2 (1990) 631–661.
- [GV 11] D. García-Beltrán, J. A. Vallejo: *An approach to omni-Loday algebroids using quasi-derivations*. Journal of Generalized Lie Theory and Applications **5** (2011), Article ID G100801.
- [GVV 12] D. García-Beltrán, J. A. Vallejo, Yu. Vorobjev: *On Lie algebroids and Poisson algebras*. Symmetry, Integrability and Geometry: Methods and Applications SIGMA **8** (2012) 006.
- [KMS 93] I. Kolár, P. W. Michor and J. Slovák: *Natural operations in differential geometry*. Springer Verlag (Berlin-Heidelberg) 1993.
- [McK 05] K. Mackenzie: *General theory of Lie groupoids and Lie algebroids*. Cambridge UP (2005).

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