Mathematical methods for theoretical physics

LEVEL:	General
PREREQUISITES:	Linear algebra, complex analysis

Jasel Berra-Montiel
Alberto Molgado

Main topics:

1. Homology and homotopy
   1.1. Simplicial complexes
   1.2. Homology groups
   1.3. Fundamental group
   1.4. Homotopy groups
   1.5. Applications to Physics
2. Manifolds and Riemannian geometry
   2.1. Calculus on manifolds
   2.2. Vector fields and Lie derivative
   2.3. Parallel transport and covariant derivative
   2.4. Riemannian manifolds
   2.5. Curvature and torsion
   2.6. Killing vectors
3. Complex manifolds
   3.1. Calculus on complex manifolds
   3.2. Complex differential forms
   3.3. Hermitian and Kähler manifolds

3.4. Harmonic forms

4. Fiber bundles

    4.1. Tangent and cotangent bundles

    4.2. Vector and principal bundles

    4.3. Connections

    4.4. Curvature

5. Spectral theory

    5.1. Metric spaces

    5.2. Function spaces

    5.3. L^p-spaces

    5.4. Hilbert spaces

    5.5. Operators

    5.6. Spectral theory

References

- Recommended books:

    [1] M. Nakahara, *Geometry, Topology and Physics* (IOP Publishing, 2005).

    [2] C. Nash, *Topology and Geometry for Physicists* (Dover Publications, 2011).

    [3] Y. Choquet-Bruhat and C. DeWitt-Morette, *Analysis, Manifolds and Physics Part I and II* (North Holland, 2004).

- Additional readings:

    [1] C. J. Isham, *Modern Differential Geometry for Physicists* (World Scientific, 1999).

    [2] B. O'Neill, *Semi-Riemannian Geometry With Applications to Relativity* (Academic Press, 1983).

[3] G. Teschl, *Mathematical Methods in Quantum Mechanics* (American Mathematical Society, 2009).

[4] V. L. Hansen, *Functional analysis: Entering Hilbert space* (World Scientific, 2006).

General relativity

LEVEL:	General
PREREQUISITES:	Analytical mechanics, Quantum mechanics, Vector calculus

Jasel Berra-Montiel
Alberto Molgado

Main topics:

1. Manifolds and tensor fields
   1.1. Manifolds
   1.2. Tangent and cotangent spaces
   1.3. Induced maps
   1.4. Tensor fields
   1.5. Metric tensor
   1.6. Differential forms and integral theorems
2. Curvature
   2.1. Parallel transport and covariant derivative
   2.2. Curvature and torsion
   2.3. Bianchi identities
   2.4. Geodesics
   2.5. Isometries and conformal maps
3. Einstein equations
   3.1. Covariance
   3.2. General relativity
   3.3. Newtonian limit and gravitational radiation
   3.4. Massive fields

4. Solutions to gravitational equations
    4.1. Minkowski
    4.2. De Sitter and anti de Sitter
    4.3. Schwarzschild
    4.4. Reissner-Nordström
    4.5. Kerr
    4.6. Taub-NUT
    4.7. Black hole thermodynamics
5. Cosmology
    5.1. Friedmann-Lemaître-Robertson-Walker
    5.2. Big-Bang
    5.3. Bianchi classification
    5.4. Inflation
    5.5. Dark matter and dark energy

References

- Recommended books:
    [1] R. M. Wald, *General relativity* (University of Chicago Press, 1984).
    [2] A. Zee, *Einstein gravity in a nutshell* (Princeton University Press, 2013).
    [3] R. Sachs and H.-H. Wu, *General Relativity for Mathematicians* (Springer, 2013).
- Additional readings:
    [1] B. O'Neill, *Semi-Riemannian Geometry With Applications to Relativity* (Academic Press, 1983).
    [2] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, 1975).

[3] T. Eguchi, P. B. Gilkey and A. J. Hanson, *Gravitation, Gauge theories and differential geometry*, Physics Reports **66**, no. 6, 213 (1980).

[4] G. F. R. Ellis, R. Maartens and M. A. H. MacCallum, *Relativistic Cosmology* (Cambridge University Press, 2012).

Quantum Field Theory

Level:	General
Prerequisites:	Analytical mechanics, Quantum mechanics

Jasel Berra-Montiel
Alberto Molgado

Main topics:

1. Canonical quantisation
   1.1. Klein-Gordon field
   1.2. Dirac field
   1.3. Electromagnetic field
   1.4. Massive vector field
2. Path integral quantisation
   2.1. Path integral formulation of quantum mechanics
   2.2. Perturbation theory and the S-matrix
   2.3. Examples of scattering processes
   2.4. Propagators and Green's functions
   2.5. Generating functional
   2.6. Feynman rules
3. Gauge theories
   3.1. Propagators in QED
   3.2. Gauge fixing
   3.3. Non-Abelian gauge fields
   3.4. Self-energy operators and vertex functions
   3.5. Ward-Takahashi identities

4. Renormalisation
    4.1. Divergences in ϕ^4-theory
    4.2. Dimensional analysis
    4.3. Dimensional regularisation
    4.4. Renormalisation group

References

- Recommended books:

[1] L. H. Ryder, *Quantum field theory* (Cambridge University Press, 1996).

[2] M. E. Peskin and D. V. Schroeder, *An introduction to Quantum Field Theory* (Sarat Book House, 2005).

- Additional readings:

[1] T.-P. Cheng and L.-F. Li *Gauge Theory of elementary particle physics* (Oxford University Press, 1988).

[2] E. Zeidler, *Quantum Field Theory I: Basics in Mathematics and Physics: A Bridge between Mathematicians and Physicists, vol. 1* (springer, 2009).

[3] A. Zee, *Quantum field theory in a Nutshell* (Princeton University Press, 2010).

Introduction to string theory

LEVEL:	General
PREREQUISITES:	Analytical mechanics, Quantum mechanics

Jasel Berra-Montiel
Alberto Molgado

Main topics:

1. Extra dimensions
   1.1. Lorentz invariance in extra dimensions
   1.2. Compact extra dimensions
   1.3. Orbifolds
   1.4. Potentials in extra dimensions
2. Bosonic string
   2.1. Brane and string actions
   2.2. Sigma models
   2.3. Canonical quantisation
3. Conformal field theory
   3.1. Classical conformal field theory
   3.2. BRST quantisation
   3.3. Vertex operators
   3.4. Perturbative string theory
4. Supersymmetry and strings
   4.1. Boundary conditions and modes
   4.2. Supersymmetric string action
   4.3. Quantisation of the supersymmetric string

4.4. Gauge anomalies

5. Duality in string theory

   5.1. Dp-branes

   5.2. T-duality and S-duality

   5.3. AdS/CFT correspondence

References

- Recommended books:

  [1] B. Zwiebach, *A First Course in String Theory* (Cambridge University Press, 2009).

  [2] J. Polchinski, *String Theory, Volume I & II* (Cambridge University Press, 2006).

- Additional readings:

  [1] M. Green, J. H. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press, 1987).

  [2] K. Becker, M. Becker and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction* (Cambridge University Press, 2007).

  [3] P. Deligne, *Quantum Fields and Strings: A Course for Mathematicians* (American Mathematical Society, 2000).

Topics in General relativity

LEVEL:	Advanced
PREREQUISITES:	General relativity, Mathematical methods for theoretical physics

Jasel Berra-Montiel
Alberto Molgado

Main topics:

1. Lagrangian formulation
   1.1. Variational principles
   1.2. Einstein-Hilbert action
   1.3. Einstein-Palatini action
2. ADM formalism for covariant theories
   2.1. Gauss-Codazzi equations
   2.2. ADM action
   2.3. Constraint analysis
   2.4. Wheeler-DeWitt equation
3. Tetrads and spin-connection
   3.1. Tetrad formalism
   3.2. Spin-connection
   3.3. Gauge invariance
   3.4. Self-dual actions
   3.5. Matter gravity couplings
   3.6. BF actions
4. Quantum gravity

4.1. Canonical quantum gravity

4.2. Path integral quantisation

4.3. Loop quantum gravity

4.4. Quantum cosmology

References

- Recommended books:

[1] R. M. Wald, *General relativity* (University of Chicago Press, 1984).

[2] C. Kiefer, *Quantum Gravity* (Oxford University Press, 2012).

- Additional readings:

[1] T. Thiemann, *Modern Canonical Quantum General Relativity* (Cambridge University Press, 2008).

[2] R. Gambini and J. Pullin, *A first course in loop quantum gravity* (Oxford University Press, 2011).

[3] C. Rovelli, *Quantum gravity* (Cambridge University Press, 2005).

Quantisation of gauge theories

Level:	Advanced
Prerequisites:	Analytical mechanics, Quantum mechanics

Jasel Berra-Montiel
Alberto Molgado

Main topics:

1. Dirac formalism for classical constrained systems
   - 1.1. Singular Lagrangians and local symmetries
   - 1.2. Hamiltonian constrained systems
   - 1.3. Constraint classification
   - 1.4. Gauge transformations
   - 1.5. Observables
   - 1.6. Dirac bracket
   - 1.7. Generally covariant systems
2. Quantisation of gauge theories
   - 2.1. Canonical quantisation
   - 2.2. Functional integral quantisation
   - 2.3. Electromagnetic field
   - 2.4. Yang-Mills theory
3. Algebraic methods of quantisation
   - 3.1. Reduced phase space vs dirac quantisation
   - 3.2. Quantisation through algebraic observables
   - 3.3. Refined algebraic quantisation
   - 3.4. Group averaging techniques

References

- Recommended books:

[1] M. Henneauxb and C. Teitelboim,*Quantization of gauge systems* (Princeton University Press, 1992).

[2] H. Rothe and K. Rothe, *Classical and quantum dynamics of constrained Hamiltonian systems* (World Scientific, 2010).

- Additional readings:

[1] N. P. Landsman, *Mathematical Topics Between Classical and Quantum Mechanics* (Springer, 1998).

[2] T. Thiemann, *Modern Canonical Quantum General Relativity* (Cambridge University Press, 2008).

[3] A. Ashtekar and R. S. Tate, *An algebraic extension of Dirac quantization: Examples*, J. Math. Phys **35**, 6434 (1994).

[4] A. Ashtekar, J. Lewandowski, D. Marolf, J. Mourão and T. Thiemann, *Quantization of diffeomorphism invariant theories of connections with local degrees of freedom* J. Math. Phys **36**, 6456 (1995).